Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

# CALCULUS.

Conducted by **J. M. COLAW**, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

53. Proposed by **O. D. SMITH, A. M.**, Professor of Mathematics, Alabama Polytechnic Institute, Auburn, Alabama.

Solve the differential equation, $dy/dx=y(x-y)/x(x+y)$, and show that $x=y\log(xy)$.

I. Solution by **J. SCHEFFER, A. M.**, Hagerstown, Maryland; **C. W. M. BLACK**, Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts; and **P. S. BERG**, Larimore, North Dakota.

Clearing of fractions we obtain after transposing two terms

$$y(xdy+ydx)=x(ydx-xdy).$$

Dividing by $y^2$, we have

$$xdy+ydx=xy.\frac{ydx-xdy}{y^2}, \text{ or, } d(xy)=xy.d\left(\frac{x}{y}\right). \quad \therefore \frac{d(xy)}{xy}=d\left(\frac{x}{y}\right).$$

Integrating, $\log(axy)=(x)\div(y)$, whence $x=y\log(axy)$.

The result given is not general enough, the constant having been left out of consideration.

II. Solution by **W. W. LANDIS, A. M.**, Department of Mathematics and Astronomy in Dickinson College, Carlisle, Pennsylvania; **F. M. McGAW, A. M.**, Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey; **J. OWEN MAHONEY, B. E.**, Graduate Fellow and Assistant in Mathematics, Vanderbilt University, Nashville, Tennessee; **G. B. M. ZERR, A. M., Ph. D.**, Texarkana, Arkansas-Texas; and **A. H. HOLMES**, Brunswick, Maine.

Let $y=vx$, then the equation becomes

$$v+\frac{xdv}{dx}=\frac{x(1-v)}{1+v}, \text{ or } 2v^2+\frac{x(1+v)dv}{dx}=0. \quad \therefore \frac{1+v}{2v^2}dv+\frac{dx}{x}=0.$$

The variables are separable, whence

$$\frac{dv}{2v^2}+\frac{dv}{2v}+\frac{dx}{x}=0. \quad \text{Integrating, } \frac{1}{2v}=\tfrac{1}{2}\log v+\log x.$$

$\therefore$ $1/v=\log v+\log x^2=\log(vx^2)$.

$\therefore$ $x/y=\log(xy)$, or $x=y\log(xy)$, when no constant is added, or $x=y\log(xy)+Cy$ where $C$ is an arbitrary constant.

**III. Solution by M. C. STEVENS, M. A., Mathematical Department, Purdue University, Lafayette, Indiana ; HENRY HEATON, M. Sc., Atlantic, Iowa ; JOHN B. FAUGHT, A. M., Instructor in Mathematics in Indiana University, Bloomington. Indiana ; and J. C. GREGG, A. M.. Brazil, Indiana.**

Put $y=vx$ and we have

$$v+x\frac{dv}{dx}=\frac{vx^2-v^2x^2}{x^2+vx^2}=\frac{v-v^2}{1+v}=v-\frac{2v^2}{1+v}.\quad \text{Whence, } \frac{1+v}{v^2}dv+\frac{2dx}{x}=0.$$

Integrating, $-1/v+\log(vx^2)+C=0$, or $x/y=\log(xy)+C$, and $x=y\log(xy)+Cy$.

The $C$ should not be omitted unless the conditions of the question giving rise to the equation are such as to make it zero.

**IV. Solution by H. C. WHITTAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

Let $y=x^pv^q$ and substitute in the given equation and we obtain

$$\frac{dv}{dx}=\frac{(1-p)v-(1+p)x^{p-1}v^{q+1}}{q^x(1+x^{p-1}v^q}.$$

This will reduce to a simple form if we take $p=1$ and $q=-1$, giving

$$\frac{dv}{dx}=\frac{2}{x(1+v^{-1})},\text{ or } dv(1+v^{-1})=2x^{-1}dx.$$

$v+\log v=\log x^2$ ; $x/y+\log(x/y)=\log x^2$.
$x/y=\log x^2-\log(x/y)=\log(xy)$, whence $x=y\log(xy)$.

**54. Proposed by J. SCHEFFER, A. M., Hagerstown, Maryland.**

A certain solid has a square, side$=a$, for its base, and all parallel sections are squares, the two sections through the middle points of the opposite side of the square are semi-circles, however. Find surface, volume, and center of gravity of each.

**I. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.**

The length of a side of a parallel section distant $x$ from the base is $(a^2-4x^2)^{\frac{1}{2}}$. If $dx$ be the distance between two parallel sections, the distance between two corresponding sides is $adx/(a^2-4x^2)^{\frac{1}{2}}$. Hence the surface

$$S=4\int_0^{\frac{1}{2}a}adx=2a^2\text{ ; the volume } V=\int_0^{\frac{1}{2}a}(a^2-4x^2)dx=\tfrac{1}{3}a^3\text{ ;}$$

the distance of the center of gravity of the surface from the base is

$$\frac{1}{2a^2}\int_0^{\frac{1}{2}a}axdx=\tfrac{1}{8}a\text{ ;}$$